*Total number of printed pages – 7*

**63 (FY)SEM–5/MAJ/PHYMAJ3014**

**2025**

# PHYSICS

## ( MAJOR )

Paper : PHYMAJ3014

**( Classical Mechanics )**

*Full Marks : 70*

*Pass Marks : 28*

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Choose the correct answer : $1\times6=6$

   *(a)* Number of degrees of freedom of a rigid body in 3 dimensional (3D) space is:

   *(i)* 3

   *(ii)* 4

   *(iii)* 6

   *(iv)* 9

(b) D'Alembert's principle is based on:

(i) Principle of least action

(ii) Principle of virtual work

(iii) Principle of infinitesimal transformations

(iv) Principle of Euler's transformations

(c) Lagrange's equation for *non-conservative* system is:

(i) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=Q_j$

(ii) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial q_j}=-Q_j$

(iii) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=0$

(iv) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial q_j}=0$



(d) Holonomic constraints are:

(i) Independent of time

(ii) Explicitly depends on time

(iii) Independent of velocity

(iv) Explicitly depends on velocity

(e) For two functions $F$ and $G$, depends upon the position coordinate $q_k$ and momentum coordinate $p_k$, which of the following is correct:

(i) $[F,G]=[F,F]$

(ii) $[F,G]=[G,F]$

(iii) $[F,G]=-[F,G]$

(iv) $[F,G]=-[G,F]$

(f) If the potential energy function is given by $V(x)=x^2-x^4$. The system has the stable equilibrium at

(i) At $x=0$

(ii) At $x=\sqrt{\frac{2}{3}}$

(iii) At $x = -\sqrt{\frac{2}{3}}$

(iv) At $x = \frac{2}{3}$

2. Answer the following questions : ***(any five)*** 2×5=10

(a) Define phase space and write its importance in dynamical system.

(b) State and define canonical momentum.

(c) Write *two* advantages of using Hamiltonian dynamics over Lagrangian dynamics.

(d) A charged particle with charge "$q$" is under the influence of crossed electric and magnetic field. Write the equation of motion.

(e) Lagrangian of a moving particle is given by $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$. Find the generalized momentum corresponding to $r$ and $\theta$.

(f) Prove that Poisson bracket of two constants of motion is itself a constants of motion.



(g) What do you mean by normal modes of a system ?

3. Answer the following questions : ***(any six)*** 5×6=30

(a) Derive the equation of motion of a simple harmonic oscillator using Lagrange's equation.

(b) For a simple pendulum with mass '$m$', deduce the Hamiltonian and hence the equation of motion.

(c) For a particle moving in one dimension whose Lagrangian is given by $L = \frac{\dot{x}^2}{2x} - V(x)$. Prove that its Hamiltonian is given by $H = \frac{1}{2}xp^2 + V(x)$.

(d) Show that the transformations $P = \frac{1}{2}(p^2 + q^2), Q = tan^{-1}\left(\frac{q}{p}\right)$ is canonical.

(e) What is a rigid body? Obtain Euler's equations of motion for a rigid body.

(f) Using Hamilton's equation of motion, describe the motion of the particle of mass "$m$", moving near the surface of the Earth under Earth's uniform gravitational field.

(g) Find the kinetic energy of a rigid body rotating about a fixed point.

(h) Deduce Hamilton's principle from D'Alembert principle.

(i) Show that the fundamental Poisson bracket is invariant under canonical transformation.

4. Answer the following questions: ***(any two)*** 12×2=24

(a) State and prove Liouvillie's theorem. 2+10=12

(b) Define and distinguish between stable and unstable equilibrium using a potential energy curve. Two beads of masses $m$ and $2m$ are connected to a *fixed wall with two springs of spring constants* $k$ *and* $2k$ over a massless, *frictionless horizontal* wire. Construct the Lagrangian. Find the eigenfrequencies of small amplitude oscillations of the system. 2+2+2+6=12



(c) (i) Derive the Lagrange equation of motion for a conservative system. 6

(ii) Show that the generalized momentum corresponding to a cyclic coordinate remains conserved. 6

(d) For *any three* function $F(q_k p_k)$, $G(q_k p_k)$ and $K(q_k p_k)$, Prove that

$$[F,[G,K]]+[G,[K,F]]+[K,[F,G]]=0$$ 12

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